September 23, 2015

NIKHEF 2015-026
DAMTP-2015-45

Quantum corrections in Higgs inflation: the Standard Model case

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ABSTRACT

We compute the one-loop renormalization group equations for Standard Model Higgs inflation. The calculation is done in the Einstein frame, using a covariant formalism for the multi-field system. All counterterms, and thus the betafunctions, can be extracted from the radiative corrections to the two-point functions; the calculation of higher n-point functions then serves as a consistency check of the approach. We find that the theory is renormalizable in the effective field theory sense in the small, mid and large field regime. In the large field regime our results differ slightly from those found in the literature, due to a different treatment of the Goldstone bosons.

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1 Introduction

In Higgs inflation the Higgs field of the Standard Model (SM) is coupled non-minimally to gravity [1, 2, 3, 4, 5, 6]. Apart from this single non-minimal coupling, no new physics is needed to describe inflation and the subsequent period of reheating, and the theory seems to be extremely predictive. However, this presupposes the parameters of the theory at high and low scale are related by renormalization group (RG) flow. The betafunctions in the low scale regime are the usual SM ones; in this paper we calculate the RG flow in both the mid field and the high scale (inflationary) regime.

The idea of using the Higgs as the inflaton is an attractive one, not least because it allows one to connect collider observables with measurements of the early universe. Since its inception the model itself has come under a lot of scrutiny and criticism. First, unitarity is lost at high energies and the perturbative theory can only be trusted for energies below the unitarity cutoff [7, 8, 9, 10, 11]. Although it is uncertain how to interpret this result as the cutoff is field dependent (according to [6, 12, 13, 14], all relevant physical scales are always below the unitarity bound), it is clear that any new physics living at this scale may affect the inflationary predictions [11, 15]. Second, given the currently measured central values for the top and Higgs mass, the Higgs potential becomes unstable around 10^{11} GeV, which would be disastrous for Higgs inflation. However, the verdict is not yet out, as it only takes $2-3\sigma$ deviations to push the instability bound all the way to the Planck scale [16, 17, 18, 19, 20, 21] (in the very recent note [22] absolute stability of the electroweak vacuum is reported to be excluded by only 1.3 σ).

Even though these claims are still debated, and SM Higgs inflation may still be alive, it is worth noting that constraints may be avoided in modified set-ups with an extended Higgs sector. Our results apply for large non-minimal coupling, but apart from that they are equally applicable to the various implementations of Higgs inflation.

The renormalization group equations (RGEs) in Higgs inflation have been derived by several groups [4, 5, 23, 24, 25, 26], but they differ in details. The main source of disagreement comes from the choice of frame, and the treatment of the Higgs sector (Does the Higgs decouple from all fields? And the Goldstone bosons?). In previous work [27, 28] we have shown that the Jordan and Einstein frame describe exactly the same physics, and that any difference stems from an erroneous comparison of quantities defined in different frames. In this work we will work in the Einstein frame. Although dimensional analysis indicates that some of the Goldstone boson (GB)-loop corrections are large, and seem to spoil renormalization, gauge symmetry kicks in leading to cancellations of these large contributions. We find Higgs inflation is renormalizable in the effective field theory (EFT) sense [1], and for energies below the unitarity cutoff.

The small-field regime of Higgs inflation is where $\phi_0 \ll m_{\rm p}/\xi$, with ϕ_0 the value of the background Higgs field, ξ the non-minimal Higgs-gravity coupling which is of order 10^4 (well below experimental bounds [29, 30]), and $m_{\rm p}$ the Planck mass. In this regime the theory is effectively like the SM and therefore renormalizable in the EFT sense. In the large field regime ($\phi_0 \gg m_{\rm p}/\sqrt{\xi}$, corresponding to inflation) the potential has an approximate shift symmetry,

[1] Higgs inflation is non-renormalizable as the field space metric and potential are non-polynomial. But this does not exclude that the theory is renormalizable in the EFT sense (as is the case in the IR). Our demands are that in the large and mid field regime the theory can be expanded in a small parameter δ, and that all loop corrections can be absorbed in counterterms order by order. Truncating the theory at some finite order in δ gives a renormalizable EFT with a finite number of counterterms.

which restricts the form of the loop corrections. As a result, all one-loop corrections can be absorbed in the parameters of the classical theory, and the EFT is renormalizable. Somewhat surprisingly, we find the same in the mid-field regime ($m_{\rm p}/\xi < \phi_0 < m_{\rm p}/\sqrt{\xi}$), even though it is far away from both an IR fixed point and the region in which the shift symmetry applies.

In [27] we have studied the renormalization of the non-minimally coupled Higgs field in isolation, without any gauge or fermion fields, and our findings were in line with the literature. In this work we want to extend this previous analysis to the full SM. At first glance this does not seem to be problematic. Due to the non-minimal coupling to gravity, the coupling of the radial Higgs to both gauge field and fermions is suppressed in the large field regime. One can simply neglect all diagrams with these couplings. For example, loop diagrams with a fermion or gauge boson loop always dominate over the corresponding diagram with a Higgs loop. Effectively the Higgs decouples from the theory. However, the situation for the Goldstone bosons (GBs) is more complex: their coupling to the gauge fields is also suppressed, while the GB-fermion coupling is not. Upon going to unitary gauge, this corresponds to a coupling of the fermion to the longitudinal polarization of the gauge fields, and both the transverse and longitudinal polarizations couple with the usual SM strength to the fermions.

All calculations are performed in the Einstein frame. For a discussion of the equivalence of Einstein and Jordan frame, see [27, 28]. One of the main complications in the calculation is that after transforming to the Einstein frame one ends up with non-canonical kinetic terms for the Higgs and Goldstone field. Due to the nonzero curvature of the field space, it is impossible to make a field transformation that brings the kinetic terms to their canonical form. Our approach here is to expand the action around a large classical background value for the inflaton field, and use the formalism of [31, 32, 33] so that this background expansion can be done maintaining covariance in the field space metric.

In our calculation we have neglected the time-dependence of the background field, as well as FLRW corrections and the backreaction from gravity; we argue that these corrections are at most subleading. (The inclusion of gravity corrections to the Higgs part of the theory has been addressed, in a covariant way, in [13].) Moreover, we are neglecting higher order kinetic terms by evaluating the field metric on the background. It would be an interesting but equally challenging task to develop a framework that can get around this latter limitation.

Our main results are the SM Higgs inflation RGEs in the three regimes, where we included only the top-Yukawa coupling y_t. We find that Higgs/GB self-interactions and Higgs-fermion-interactions (but not GB-fermion) can be neglected in the mid and large field regime; Higgs/GB-gauge interactions decouple in the large field regime. This gives the following

betafunctions:

$$
\begin{aligned}
(4\pi)^2\beta_\lambda &= 24\lambda^2 s + A + (4\pi)^2\cdot 4\lambda\gamma_\phi \\
(4\pi)^2\gamma_\phi &= -\frac{s}{4}(3g_1^2+9g_2^2)+3y_t^2 \\
(4\pi)^2\beta_{g_3} &= -7g_3^3, \\
(4\pi)^2\beta_{g_2} &= -\frac{(20-s)}{6}g_2^3, \\
(4\pi)^2\beta_{g_1} &= \frac{(40+s)}{6}g_1^3 \\
(4\pi)^2\beta_{y_t} &= \left[\frac{3}{2}sy_t^3-\left(\frac{2}{3}g_1^2+8g_3^2\right)y_t\right]+(4\pi)^2\cdot\gamma_\phi y_t \\
(4\pi)^2\beta_\xi\big|_{\text{mid,large}} &= (4\pi)^2\cdot 2\gamma_\phi\xi
\end{aligned} \tag{1}
$$

with $A=(3/8)(2g_2^4+(g_2^2+g_1^2)^2)-6y_t^4$ and

$$
s=\begin{cases}1, & \text{small,}\\ 0, & \text{mid, large.}\end{cases} \tag{2}
$$

These betafunctions break down at the boundary of the regimes, where the EFT expansion in a small parameter is no longer valid; this gives additional threshold corrections which we have not calculated.

All sign conventions used in this paper follow the QFT textbook by Srednicki [35], except for the sign of the Yukawa interaction terms, which is opposite to Srednicki's.

2 Higgs inflation

In this section we give a brief overview of Higgs inflation and set our notation.

2.1 Lagrangian

The Jordan frame Lagrangian[2] is (using $-+++$ metric signature)

$$
\mathcal{L}^J=\sqrt{-g^J}\left[-\frac{1}{2}m_\text{p}^2\left(1+\frac{2\xi\Phi^\dagger\Phi}{m_\text{p}^2}\right)R[g^J]+\mathcal{L}_\text{SM}^J\right], \tag{3}
$$

with

$$
\begin{aligned}
\mathcal{L}_\text{SM}^J = & -\frac{1}{4}(f_{\mu\nu}^a)^2-\frac{1}{4}(F_{\mu\nu}^a)^2-\frac{1}{4}B_{\mu\nu}^2-(D_\mu\Phi)^\dagger(D^\mu\Phi)-\lambda(\Phi^\dagger\Phi-v^2/2)^2 \\
& +\bar{Q}_L(i\not{D})Q_L+\bar{u}_R(i\not{D})u_R+\bar{d}_R(i\not{D})d_R-(y_d\bar{Q}_L\cdot\Phi d_R+y_u\bar{u}_R(i\sigma^2)\Phi^\dagger Q_L+\text{h.c}),
\end{aligned} \tag{4}
$$

where $Q_L=(u\ d)_L^\top$. Further $f_{\mu\nu}^a, F_{\mu\nu}^a, B_{\mu\nu}$ are the SU(3), SU(2) and U(1) field strengths respectively. The Higgs field is SU(2) complex doublet, which we parameterize

$$
\Phi=\frac{1}{\sqrt{2}}\begin{pmatrix}\varphi^+\\ \phi_0+\varphi+i\theta_3\end{pmatrix}, \tag{5}
$$

[2] For considerations about initial conditions for Higgs inflation, and the possible inclusion of a R^2-term, see [34]

with ϕ_0 the classical background, φ the Higgs field and $\varphi^+ = \theta_1 + i\theta_2, \theta_3$ the GBs. The covariant derivative acts on the Higgs field and fermions as

$$\begin{aligned} D_\mu \Phi &= \left(\partial_\mu - ig_2 A^a_\mu \tau^a - iY_\phi g_1 B_\mu\right)\Phi, \\ D_\mu Q_L &= (\partial_\mu - ig_3 f^a_\mu t^a - ig_2 A^a_\mu \tau^a - iY_Q g_1 B_\mu) Q_L, \\ D_\mu u_R &= (\partial_\mu - ig_3 f^a_\mu t^a - iY_u g_1 B_\mu) u_R, \end{aligned} \tag{6}$$

with $\tau^a = \sigma/2$ for the spinor representation. The hypercharges are $Y_\phi = 1/2$, $Y_Q = 1/6$ and $Y_u = 2/3$. At leading order, the only fermion that matters to find the running of the SM couplings is the top quark.

We reach the Einstein frame after a conformal transformation: $g^E_{\mu\nu} = \Omega^2 g^J_{\mu\nu}$, with

$$\Omega^2 = \left(1 + \frac{2\xi\Phi^\dagger\Phi}{m_{\rm p}^2}\right). \tag{7}$$

The Einstein frame Lagrangian becomes

$$\mathcal{L}^E = \sqrt{-g^E}\left[-\frac{1}{2}m_{\rm p}^2 R[g^E] + \mathcal{L}^E_{\rm mat}\right]. \tag{8}$$

We neglect the expansion of the universe, and take a Minkowski metric.

The gauge kinetic terms are conformally invariant. The fermionic kinetic terms can be made canonical via a rescaling $\psi^E = \psi/\Omega^{3/2}$; the net effect is then a rescaling of the Yukawa interaction. All non-trivial effects of the non-minimal coupling are in the Higgs sector.

$$\begin{aligned} \mathcal{L}^E_{\rm mat} = &-\frac{1}{4}(f^a_{\mu\nu})^2 - \frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{4}B^2_{\mu\nu} - \frac{1}{\Omega^2}(D_\mu\Phi)^\dagger(D^\mu\Phi) - \frac{3\xi^2}{m_{\rm p}^2\Omega^4}\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi) \\ &+ \bar{Q}^E_L(i\not{D})Q^E_L + \bar{u}^E_R(i\not{D})u^E_R + \bar{d}^E_R(i\not{D})d^E_R \\ &- \frac{\lambda}{\Omega^4}(\Phi^\dagger\Phi - v^2/2)^2 - (\frac{y_d}{\Omega}\bar{Q}^E_L\cdot\Phi d^E_R + \frac{y_u}{\Omega}\bar{u}^E_R(i\sigma^2)\Phi^\dagger Q^E_L + \text{h.c.}) \end{aligned} \tag{9}$$

The Higgs kinetic term is non-minimal. Let $\phi^i = \{\phi_R = \phi_0 + \varphi, \theta_i\}$ run over the Higgs field and Goldstone bosons. Then the metric in field space in component form is

$$\mathcal{L}^E_{\rm mat} \supset -\frac{1}{2}\gamma_{ij}\partial\phi_i\partial\phi_j = -\frac{1}{2}\left[\frac{\delta_{ij}}{\Omega^2} + \frac{6\xi^2}{m_{\rm p}^2\Omega^4}\chi_i\chi_j\right]\partial\phi_i\partial\phi_j. \tag{10}$$

The curvature on field space $R[\gamma_{ij}] \neq 0$ (this point was made, and further generalized, in [36], and the kinetic terms cannot be diagonalized. At most one can diagonalize the quadratic kinetic terms at one specific point in field space.

Consider the electroweak sector. For the gauge bosons the kinetic terms remain canonical in the Einstein frame. As far as the quadratic action is concerned the action for the massive gauge bosons and Goldstone bosons is simply three times the action of a U(1) theory. To see this explicitly, consider the Higgs kinetic terms

$$\begin{aligned} \mathcal{L}_{\rm higgs} &\supset -\frac{1}{\Omega^2}(D_\mu\Phi)^\dagger(D^\mu\Phi) \\ &= -\frac{1}{2\Omega^2}\left[\partial_\mu\varphi\partial^\mu\varphi + \sum_{a=1}^{3}(\partial_\mu\theta_a\partial^\mu\theta_a - 2g_a A^a_\mu(\phi\partial^\mu\theta_a - \theta_a\partial^\mu\phi) + g_a^2\phi^2 A^a_\mu A^\mu_a) + ...\right]. \end{aligned} \tag{11}$$

The gauge boson mass eigenstates are $\{A_1, A_2, Z, A_\gamma\}$ with

$$Z = \frac{1}{\sqrt{g_2^2+g_1^2}}(g_2 A_3 - g_1 B), \quad A_\gamma = \frac{1}{\sqrt{g_2^2+g_1^2}}(g_1 A_3 + g_2 B), \tag{12}$$

and couplings

$$g_a = \frac{1}{2} \times \left\{ g_2, g_2, \sqrt{g_2^2+g_1^2}, 0 \right\}. \tag{13}$$

This corresponds to three massive and one massless field. Note that we took the mass eigenstates as real gauge fields, and used the real and imaginary parts of W_+, rather than the complex states $W_\pm$.

From now on we will work in the Einstein frame. For convenience we drop the superscript E, and work in Planck units $m_{\rm p} = 1$.

2.2 Three regimes

Higgs inflation is non-renormalizable as the field space metric and potential are non-polynomial. But this does not exclude that the theory is renormalizable in the EFT sense over a limited field space. Our demands are that in a given field regime the theory can be expanded in a small parameter δ, and that all loop corrections can be absorbed in counterterms order by order. Truncating the theory at some finite order in δ gives a renormalizable EFT with a finite number of counterterms.

Small field regime The small field regime corresponds to $\delta_s \equiv \xi\phi_0 \ll 1$. To leading order in the expansion parameter δ_s, the Lagrangian reduces to the SM Lagrangian.

Mid field regime The mid field regime corresponds to $1/\xi < \phi_0 < 1/\sqrt{\xi}$. In this regime we rescale $\xi \to \delta_m^{-2}\xi$ and $\phi_0 \to \delta_m^{3/2}\phi_0$, such that both $\xi\phi_0^2 \propto \delta_m$ and $1/(\xi\phi_0)^2 \propto \delta_m$, and use δ_m as our expansion parameter. (We should admit that formally this expansion can only be trusted in the middle of this regime.)

Large field regime Inflation takes place for field values $\delta_l \equiv 1/(\xi\phi_0^2) \ll 1$. The expansion in δ is equivalent to an expansion in slow-roll parameters, since $\eta = \mathcal{O}(\delta)$ and $\epsilon = \mathcal{O}(\delta^2)$.

3 Covariant formalism and counterterms

We want to investigate how the loop corrections and counterterms change in the small, mid and large field regime. For simplicity, we first focus on a U(1) Abelian Higgs model coupled to a left- and right-handed fermion. The generalization to the full SM Higgs inflation is postponed till sec. 5.1. This way the effects of the non-minimal coupling can be studied in a simple set-up, without all intricacies of the chiral SM. Another advantage of the U(1) model is that gauge invariance and the Ward identities assure that many counterterms are independent of the gauge choice, which makes it easier to check the calculation.

3.1 Lagrangian in covariant fields

This subsection reviews the covariant formalism introduced in [31] and further worked out in [32, 33]. Given the curvature of field space, it is very convenient to adopt an approach that maintains the covariance of the equations.

For a U(1) theory with a complex Higgs field and a left- and right-handed Weyl fermion the Einstein frame matter Lagrangian is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\gamma_{ab}\partial_\mu\phi^a\partial^\mu\phi^b + i\bar{\psi}\partial\!\!\!/\psi - V(\phi^a) - \bar{\psi}F(\phi^a)\psi \\ &- gA(G^\theta\partial\phi - G^\phi\partial\theta) - \frac{1}{2}g^2A^2G + \left(gq_L\bar{\psi}A\!\!\!/P_L\psi + gq_R\bar{\psi}A\!\!\!/P_R\psi\right),\end{aligned} \tag{14}$$

with

$$\begin{aligned} V(\phi^a) &= \frac{\lambda}{4}\frac{|\phi_0+\varphi+i\theta|^4}{\Omega^4}, \qquad F(\phi^a) = \frac{y}{\sqrt{2}}\frac{\phi_0+\varphi+i\gamma^5\theta}{\Omega}, \\ G^\phi &= \frac{\phi}{\Omega^2}, \qquad G^\theta = \frac{\theta}{\Omega^2}, \qquad G = \frac{(\phi^2+\theta^2)}{\Omega^2}. \end{aligned} \tag{15}$$

Now expand the Lagrangian around the background $\phi^a = (\phi_0(t)+\varphi(x,t),\theta(x,t))$. The fluctuation fields $\delta\phi^a = (\varphi,\theta)$ are not in the tangent space at ϕ^a_0, and therefore do not transform as a tensor. We are led to introduce the covariant fluctuation $Q^a = (h,\chi)$, which is related to $\delta\phi^a$ via

$$\phi^a = Q^a - \frac{1}{2!}\Gamma^a_{bc}Q^bQ^c + \frac{1}{3!}\left(\Gamma^a_{be}\Gamma^e_{cd} - \Gamma^a_{bc,d}\right)Q^bQ^cQ^d + \ldots \tag{16}$$

This is the notation we will use throughout this paper: (φ,θ) are the fluctuations of the original Jordan frame field (with ϕ_0 the classical background field), and (h,χ) are the covariant fields. Further we define the covariant time derivative

$$D_t = \frac{\mathrm{d}\phi^a}{\mathrm{d}t}\nabla_a. \tag{17}$$

Note that in the limit $\dot{\phi}_0 = 0$ this reduces to the usual derivative $D_t = \partial_t$.

Now we can expand the action in covariant fluctuations. We neglect FLRW corrections and the backreaction from gravity, as well as the time-dependence of the background field ϕ_0; we come back to this in Sec. 3.5. The result for the interaction Lagrangian is

$$\begin{aligned}\mathcal{L}_{\rm int} = &-\left(V + V_{;a}Q^a + \frac{1}{2!}V_{;ab}Q^aQ^b + \ldots\right) - \bar{\psi}\left(F + F_{;a}Q^a + \frac{1}{2!}F_{;ab}Q^aQ^b + \ldots\right)\psi \\ &- gA\partial h(G^\theta_{;a}Q^a + \frac{1}{2!}G^\theta_{;ab}Q^aQ^b + \ldots) + gA\partial\chi(G^\phi_{;a}Q^a + \frac{1}{2!}G^\phi_{;ab}Q^aQ^b + \ldots) \\ &- \frac{1}{2}g^2A^2(G_{;a}Q^a + \frac{1}{2!}G_{;ab}Q^aQ^b + \ldots) + \left(gq_L\bar{\psi}A\!\!\!/P_L\psi + gq_R\bar{\psi}A\!\!\!/P_R\psi\right).\end{aligned} \tag{18}$$

All coefficients are evaluated on the background. The subscript with a semi-colon denotes the covariant derivative.

We just found the Lagrangian for the covariant fields by Taylor expanding using covariant derivatives. An equivalent way of deriving the same Lagrangian is solving the relation $\phi^i(Q^j)$

(16) explicitly, substituting in the Lagrangian (14), [3] and then Taylor expand in the fields Q^i (using partial derivatives). This point of view will be useful when defining the counterterms in the next section. Here we just give the explicit form of (16) relating the original Langragian fields (φ, θ) to the covariant fields (h, χ):

$$\begin{aligned}
\varphi &= \left(h + \frac{(h^2-\chi^2)}{2\phi_0} + ..\right) - \frac{1}{\xi}\left(\frac{h^2}{\phi_0^3} + \frac{h^3}{3\phi_0^4} + ...\right) + \frac{1}{\xi^2}\left(\frac{h^2+\chi^2}{12\phi_0^3} + ...\right). \\
\theta &= \left(\chi + \frac{h\chi}{\phi_0} + ..\right) - \frac{1}{\xi}\left(\frac{h\chi}{\phi_0^3} + \frac{4h^2\chi}{3\phi_0^4} + ...\right) + \frac{1}{\xi^2}\left(\frac{h\chi}{\phi_0^5} + \frac{h^2\chi}{12\phi_0^4} + ...\right).
\end{aligned} \tag{19}$$

We checked that substituting this in the Lagrangian and expanding, we indeed retrieve (18).

3.2 Gauge fixing

We have to add a gauge fixing and ghost Lagrangian, which can also be expanded in covariant fields.

We fix the gauge via

$$\mathcal{L}^E_{\rm GF} = -\frac{1}{2\xi_G}\left(\partial^\mu A_\mu - gG^\phi(\phi_0)\xi_G\theta\right)^2. \tag{20}$$

This removes the quadratic $A\partial\theta$ couplings from the Lagrangian. In the small field regime $\Omega_0 \equiv \Omega(\phi_0) = 1$ and we retrieve the standard R_ξ-gauge. We choose to write the gauge fixing term in terms of the Jordan frame fields (as opposed to the covariant fields) as these have a well defined gauge transformation.

We work in Landau gauge $\xi_G = 0$. Then the ghost field decouples

$$\mathcal{L}^E_{\rm FP}\big|_{\xi_G=0} = -\partial_\mu \bar{c}\partial^\mu c. \tag{21}$$

3.3 Feynman rules

Now we can derive the Feynman rules from the above action. First we define the effective couplings

$$\begin{aligned}
\mathcal{L}_{\rm int} = &-\lambda_{mhn\chi}h^m\chi^n - y_{mhn\chi}h^m\chi^n\bar{\psi}(i\gamma^5)^\alpha\psi - (g_{A\partial hmhn\chi}\partial h - g_{A\partial\chi mhn\chi}\partial\chi)Ah^m\chi^n \\
&- g_{2Amhn\chi}A^2h^m\chi^n + g_L\bar{\psi}\not{A}P_L\psi + g_R\bar{\psi}\not{A}P_R\psi
\end{aligned} \tag{22}$$

with $\alpha = 1$ if the number $n =$ odd, and $\alpha = 0$ otherwise (signs are absorbed in the couplings). All interactions are defined with a minus sign (the only exception is for one of the derivative interactions and the fermion-gauge interaction), and without numerical factors. This means that for a vertex with m h-fields and n χ-fields and with or without fermion/gauge lines we have, respectively:

$$\begin{aligned}
V^{(mhn\chi)} &= (-i)m!n!\lambda_{mhn\chi}, \\
V^{(mhn\chi 2\psi)} &= (-i)m!n!y_{m\phi n\chi}(i\gamma^5)^\alpha, \\
V^{(mhn\chi 2A)} &= (-i)2!m!n!g_{2Amhn\chi}.
\end{aligned} \tag{23}$$

[3] Equivalently, and probably more easily, the expansion $\phi^i(Q^i)$ can be found by $\phi^i = \phi^i_{;a}Q^a + \frac{1}{2!}\phi^i_{;ab}Q^aQ^b + ...$

For the derivative interaction we get

$$V^{(A\partial hmhn\chi)} = -ig_{(A\partial hmhn\chi)}(-ik^\mu), \quad V^{(A\partial\chi mhn\chi)} = ig_{(A\partial\chi mhn\chi)}(-ik^\mu), \tag{24}$$

with k the momentum running through the vertex. The fermion, scalar and gauge propagators are given by:

$$\begin{aligned}
-iD_\psi(k) &= \frac{-i(-\not{k} + m_\psi)}{k^2 + m_\psi^2 - i\epsilon}, \\
-iD_{Q^a}(k) &= \gamma^{aa}\frac{-i}{k^2 + (m^2)^a_a - i\epsilon}, \\
-iD_{\mu\nu}(k) &\overset{\xi_G=0}{=} -i\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2 + m_A^2 - i\epsilon},
\end{aligned} \tag{25}$$

with masses $m_\psi = F(\phi_0)$, $m_A = G(\phi_0)$, and $(m^2)^a_b = \gamma^{ac}V_{;cb}(\phi_0)$. The scalar mass is diagonal, which we used in the scalar propagator. The propagators are standard except for the metric factor in the scalar propagator. The ghost field decouples.

The above expansion of the Lagrangian, and corresponding Feynman rules are equally valid in all field regimes (although in the small field regime the notation is overkill). The explicit expressions for the vertices are given in Appendix A. The Higgs/GB mass and self-interactions are suppressed in the mid and large field regime, and we can neglect Higgs/GB loops in the quantum corrections. Likewise, the interaction with the gauge field is suppressed in the large field regime. The Higgs-fermion coupling is small in the mid and large field regime, but the GB-fermion coupling is not: it has the standard SM strength. This does not come as a surprise. The gauge-fermion interactions are unaffected by the non-minimal coupling. And since the GB is eaten to become the longitudinal polarization of the gauge boson, it should have the same interaction strength as the transverse polarizations. A priori it is not clear what the effects are on the betafunctions. The explicit computation is done in the next section.

3.4 Counterterms

In this section we introduce counterterms. It proves convenient, or maybe even necessary, to define the wave functions Z_i in terms of the original Jordan frame fields, rather than the covariant fields. The usual U(1) symmetry relations between the various Z_i-factors then apply.

To define the counterterms we start with the Einstein frame Lagrangian in terms of the *Jordan frame* fields (14) and rescale the bare fields (with label "b") to physical fields (without label) via

$$\begin{aligned}
&\phi_b = \sqrt{Z_\phi}\phi, \qquad &&\theta_b = \sqrt{Z_\theta}\theta, \qquad &&\psi_b = \sqrt{Z_\psi}\psi, \qquad &&A^\mu_b = \sqrt{Z_A}A^\mu, \\
&\lambda_b = Z_\lambda\lambda, &&\xi_b = Z_\xi\xi, &&y_b = Z_y y, &&g_b = Z_g g,
\end{aligned} \tag{26}$$

with $\phi = \phi_0 + \varphi$. In Landau gauge $\xi_G = 0$ the wavefunctions are $Z_\varphi = Z_{\phi_0} = Z_\theta$. We further define

$$Z_i = 1 + \delta_i. \tag{27}$$

We can then split the Lagrangian $\mathcal{L} = \mathcal{L}_{\text{renormalized}} + \mathcal{L}_{\text{ct}}$ with the counterterms proportional to δ_i. The total Lagrangian is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}(Z_{g_{ab}} g_{ab}) Z_\phi \partial\phi^a \partial\phi^b + Z_\psi \bar{\psi} i\partial\!\!\!/ \psi - \frac{1}{4} Z_A F_{\mu\nu} F^{\mu\nu} \\ &- \frac{Z_\lambda Z_\phi^2 \lambda(\phi^2+\theta^2)^2}{4(1+Z_\xi Z_\phi \xi\phi^2)^2} - \frac{Z_\psi \sqrt{Z_\phi} Z_y y}{\sqrt{2}(1+Z_\xi Z_\phi \xi \phi_R^2)} \bar{\psi}(\phi + i\theta\gamma^5)\psi \\ &- \frac{1}{(1+Z_\xi Z_\phi \xi\phi_R^2)} \left(\sqrt{Z_A} Z_\phi Z_g g A(\phi\partial\theta - \theta\partial\phi) + Z_A Z_\phi Z_g^2 \frac{1}{2} g^2 A^2 (\phi^2+\theta^2) \right) \\ &+ Z_\psi Z_g Z_A^{1/2} \left(g q_L \bar{\psi} A\!\!\!/ P_L \psi + g q_R \bar{\psi} A\!\!\!/ P_R \psi \right). \end{aligned} \tag{28}$$

This can be simplified using the U(1) Ward identity

$$Z_g = Z_A^{-1/2}. \tag{29}$$

Moreover, as we will derive in the next section, we have in the mid and large field regime (in the small field regime, ξ drops out of the Lagrangian at leading order, and no counterterm can be determined at this order)

$$Z_\xi = \frac{1}{Z_\phi}, \qquad \text{for } \phi > \frac{m_{\text{p}}}{\xi}. \tag{30}$$

This means $\Omega^2 = 1 + \xi\phi^2$ does not run (in the large field regime it means $\delta = 1/(\xi\phi_0^2) \sim \mathrm{e}^{-h_0}$ does not run). We can now use $\phi^i(Q^a)$ in (19) to rewrite the Lagrangian in terms of the covariant fields. It is then clear that all interactions arising from expanding the potential have the same renormalization factor, namely $Z_\lambda Z_\phi^2$. Similar statements can be made for the gauge and Yukawa interactions.

We can also define "composite" wavefunctions for the interactions of the covariant fields via

$$\begin{aligned}\mathcal{L} \supset &= -Z_{\lambda;mhn\chi} \lambda_{mhn\chi} h^m \chi^n - Z_{y;mhn\chi} y_{mhn\chi} h^m \chi^n \bar{\psi}(i\gamma^5)^\alpha \psi \\ &- Z_{2A;mhn\chi} g_{2Amhn\chi} A^2 h^m \bar{\chi}^n - Z_{A\partial Q^a;mhn\chi} g_{A\partial Q^a mhn\chi} A\partial Q^a h^m \bar{\chi}^n. \end{aligned} \tag{31}$$

It is straightforward to rewrite the composite wavefunctions in terms of elementary ones (26), by comparing terms in the two actions above (28,31), when both written out in covariant fields. As an explicit example, consider the potential in the large field regime; using (28) it can be expanded as

$$V = \frac{Z_\lambda}{Z_\xi^2}\frac{\lambda}{4\xi^2} + \frac{Z_\lambda}{Z_\phi Z_\xi^3}\lambda \left(-\frac{1}{\phi_0^4 \xi^3} h^2 - \frac{1}{3\phi_0^6\xi^3} h^4 + \frac{1}{12\phi_0^6\xi^5}\chi^2 - \frac{1}{108\phi_0^{10}\xi^7}\chi^4 + ... \right) \tag{32}$$

from which we read off

$$Z_{V_0} = \frac{Z_\lambda}{Z_\xi^2}, \quad Z_{\lambda 2h} = Z_{\lambda 4h} = Z_{\lambda 2\chi} = Z_{\lambda 4\chi} = \frac{Z_\lambda}{Z_\phi Z_\xi^3}. \tag{33}$$

Ignoring $\dot{\phi}_0$ corrections the Higgs kinetic terms are

$$\mathcal{L} \supset -\frac{1}{2} Z_{2h} g_{hh} (\partial h)^2 = -\frac{1}{2} Z_{2h} \frac{6}{\phi_0^2} (\partial h)^2 = -\frac{1}{2}\frac{Z_h}{Z_h}\frac{6}{\phi_0^2}(\partial h)^2. \tag{34}$$

The leading counterterm vanishes. The kinetic term for the GBs is

$$\mathcal{L} \supset -\frac{1}{2} Z_{2\chi} g_{\chi\chi} (\partial\chi)^2 = -\frac{1}{2} Z_{2\chi} \delta (\partial\chi)^2 = -\frac{1}{2} Z_h \delta (\partial\chi)^2. \tag{35}$$

And thus

$$Z_{2h} - 1 = \mathcal{O}(\delta), \qquad Z_{2\chi} = Z_\phi. \tag{36}$$

To derive these results we have used $Z_h = Z_\chi = Z_\phi$, i.e. the same counterterms for the Jordan frame and covariant fields. As we will discuss below, although this approximation is valid for the kinetic terms, it is not in general.

3.4.1 On the wavefunctions of the covariant fields

Instead of defining the wavefunctions of the Jordan frame fields, we could have tried to work with those of the covariant fields. Rather than beginning from (26), we would then introduce Z_{ϕ_0}, Z_h and Z_χ. Consider the large field regime. The potential is expanded in covariant fields as in (32). We can read off the counterterms from the parametric dependence of the various terms. For the quadratic Higgs and GB interactions we find $Z_h^{-1} Z_\xi^{-3}$ and $Z_h^{-2} Z_\xi^{-5}$ respectively, which should be equal by gauge invariance. This excludes setting $Z_h = Z_\chi = Z_\phi$ in the potential, as it gives inconsistent results.[4]

As we will now discuss, this approach breaks down for the GB interactions. We can understand where this stems from. To derive the h^2 and h^4 interactions at leading order, it is enough to only keep the first term in the expansion (19). Then taking the relevant terms in (19), and setting $Q_b^i = \sqrt{Z_Q} Q^i$ we get

$$\sqrt{Z_\phi}\varphi = \sqrt{Z_h}\left(h + \frac{(h^2 - \chi^2)}{2\phi_0} + ..\right), \tag{37}$$

$$\sqrt{Z_\phi}\theta = \sqrt{Z_\chi}\left(\chi + \frac{h\chi}{\phi_0} + ..\right). \tag{38}$$

Thus for h interactions we can take $Z_h = Z_\chi = Z_\phi$. Similarly, for the kinetic terms, only the first order expansion is needed, which is what we used above.

However, to derive the GB interactions, the leading and subleading terms cancel, and to get the correct interaction one needs to expand $\phi^i(Q^j)$ to sufficient high order in δ. In particular, one needs also to take into account the last term in the expansion in (19). But then $Z_h = Z_\chi = Z_\phi = 1/Z_\xi$ is no longer a consistent solution; it would give the inconsistent relation

$$\sqrt{Z_\phi}\varphi = \sqrt{Z_\phi}(Q + Q^2 + ...) + Z_\phi^{3/2}(Q^2 + Q^3 + ...). \tag{39}$$

Thus for GB scattering one cannot really define Z_χ, Z_h in terms of the elementary wavefunctions (26). Fortunately, this is also not necessary, because we can simply use (32).

3.5 Approximations used

Before diving into the calculation we first list here the approximations made.

[4] One could leave the counterterms Z_h, Z_χ, Z_ϕ unrelated a priori, and determine them by the requirement to absorb all (1-loop) divergencies. We tried this approach, and it fails. There is no consistent choice of counterterms that renders the theory finite.

1. We have dropped the time-dependence of the background field: $\dot{\phi}_0 \to 0$
2. We have neglected FLRW corrections and the backreaction of gravity
3. We have evaluated the field metric on the classical background.

1. In [37] we calculated the effective action in the SM regime, taking into account the rolling of the classical background field $\dot{\phi}_0$. Generalizing standard techniques to calculate the effective action to the time-dependent situation, we found the radiative corrections to both the classical potential and the kinetic terms. This allowed us to extract both the δ_λ and δ_ϕ counterterms from the effective action. We retrieved the standard results. The time-dependence does not affect the form of the counterterms. Had we done the calculation in a time-independent way, by neglecting $\dot{\phi}_0$, we would have found the same δ_λ counterterm.

In the large field regime the time-dependence enters also the kinetic terms, which are non-minimal, and it may not be obvious that we can neglect these effects. However, the large field regime is the inflationary regime, and all time-dependent corrections are slow roll suppressed. Working at leading order in the expansion parameter, as we do, they can be neglected.

2. In [27, 38] we calculated the effective action in the SM regime, in a FLRW background. We showed that when working in the *Einstein frame*, the backreaction from gravity can be neglected. The reason is that the corrections are of the order of the slow roll parameter $\epsilon \sim \delta^2$, which are small compared to $\eta \sim \delta$ and thus can be neglected at leading order.

Doing the calculation in a FLRW background will give order $\mathcal{O}(H^2)$ corrections to the scalar masses, to the Higgs and GB mass in our case. However, these masses only appear in diagrams with a Higgs and GB in the loop, which thus also involve suppressed GB/Higgs couplings. There is however one diagram that becomes of leading order in the δ-expansion, which is the last term of (55), giving the GB loop correction to the fermion propagator. Nevertheless, this diagram is still suppressed by $1/\xi$. Hence, to be really sure that FLRW corrections will not affect our results we have to work in the large $\xi \gg 1$ limit.

3. The kinetic terms for the GB/Higgs field are of the form

$$\mathcal{L} \supset -\frac{1}{2}\gamma_{ij}\partial Q^i \partial Q^j = -\frac{1}{2}\gamma_{ij}(\phi_0)\partial Q^i \partial Q^j + ... \tag{40}$$

where we have expanded the field space metric around the background. The first term is quadratic and determines the structure of the propagators. The higher order terms, denoted by the ellipses above, can then be treated as additional interaction terms. It is hard to systematically take into account the effects of higher order interactions, and we have neglected them in our calculations in the next section. Unfortunately, it seems that for at least one diagram this is not a good approximation, as we discuss in section 4.5.

4 One-loop corrections

To derive the one-loop betafunctions for the gauge, Yukawa and Higgs interactions, g, y, λ, it is enough to calculate the corrections to the gauge, fermion and scalar propagator, which is what we will do in this section. We will also compute corrections to 3 and 4-point interactions. These will serve as consistency checks on the result, which provide further checks on the validity of our approximations (discussed in the previous section). Another consistency check is the comparison with the Coleman-Weinberg effective action [39].

No field independent counterterms can be defined for the whole regime, but it may be possible to define renormalizable EFTs in the three different regimes. Then the hope is that the threshold corrections in patching them together are small. To find the result in a given regime, we plug in the explicit form of the couplings expanded in the expansion parameter valid in this regime. The expansion parameters were defined in subsection 2.2; the explicit form of the couplings can be found in Appendix A.

4.1 Coleman-Weinberg effective action

The Coleman-Weinberg calculation for a dynamical background field has been performed in [37]. From this we can extract Z_{V_0}, Z_{2h}, which should be consistent with the loop corrections to the Higgs/GB propagator and self-scattering. The effective action gets contributions from the bosonic, mixed and fermionic loops respectively, and is for $\xi_G = 0$

$$\Gamma_{\mathrm{CW}} = \frac{1}{32\pi^2\epsilon}\left[m_h^4 + m_\theta^4 + 3m_A^4 - 4m_f^4 + \frac{3}{2}m_{A\theta}^4 + 4m_f\ddot{m}_f\right], \tag{41}$$

with $m_{A\theta}^2 = -2g\dot{\phi}_0/\Omega_0^2$. Adding classical and all one-loop contributions gives

$$\begin{aligned}\Gamma = {} & \frac{1}{2}\gamma_{hh}\dot{\phi}_0^2\left[-Z_{2h} + \frac{1}{8\pi^2\epsilon}\left(\frac{3g^2}{1+\xi\phi_0^2(1+6\xi)} - \frac{y^2}{\Omega_0^2(1+\xi\phi_0^2(1+6\xi))}\right)\right] \\ & + \frac{\lambda\phi_0^4}{4\Omega_0^4}\left[-Z_{V_0} + \frac{1}{8\pi^2\epsilon}\left(\lambda s(\phi_0) + 3\frac{g^4}{\lambda} - \frac{y^4}{\lambda}\right)\right],\end{aligned} \tag{42}$$

with

$$s(\phi_0) = \frac{\left(\phi_0^2\xi\left(1-2\xi\left(\phi_0^2(6\xi+1)-6\right)\right)+3\right)^2}{\Omega_0^4\left(\phi_0^2\xi(6\xi+1)+1\right)^4} + \frac{1}{\Omega_0^4\left(\phi_0^2\xi(6\xi+1)+1\right)^2}. \tag{43}$$

It is clear that no field-independent counterterms can be defined over the whole regime. Expanding the corrections in the respective regimes we find (still applying the notation $Z_i = 1+\delta_i$)

$$\delta_{V_0} = \frac{1}{8\pi^2\epsilon}\left[10s\lambda + 3\frac{g^4}{\lambda} - \frac{y^4}{\lambda}\right], \tag{44}$$

where we used notation (2). Note that $\delta_{V_0} = \delta_\lambda + 2\delta_\phi$ in the small and mid field regime, but $\delta_{V_0} = \delta_\lambda - 2\delta_\xi$ for large field. As we will see, consistency with Higgs/GB n-point functions requires $\delta_\phi = -\delta_\xi$, as in (30), and thus δ_{V_0} constrains the same elementary counterterms in the whole regime.

Furthermore, we find

$$\delta_{2h} = \frac{1}{8\pi^2\epsilon}\left[s3g^2 - sy^2 + \mathcal{O}(\delta)\right]. \tag{45}$$

In the large field regime $\delta_{2h} = \mathcal{O}(\delta)$ is a consistency check, but does not put any constraints on the elementary counterterms (26). In the mid field regime we find $\delta_{2h} = 2(\delta_\phi + \delta_\xi) = \mathcal{O}(\delta)$, and thus to lowest order (30) is satisfied. In the small field regime $\delta_{2h} = \delta_\phi$, and we find an answer consistent with (50) below.

4.2 Higgs/GB interactions

We start with the corrections to the Higgs propagator. Compared to the standard small-field calculation, the fermion loop is different because of the presence of new fermion-Higgs/GB couplings. The gauge loop proceeds as in the small field regime, with the only exception that the diagram with derivative interactions cancels (at first order in the expansion parameter) as in the large field regime $g_{A\partial h\chi} = -g_{A\partial\chi h}$ have opposite sign, instead of being equal. The result for the counterterm, fermion and gauge, mixed gauge-GB and Higgs/GB loops is

$$\Pi^h = -\delta_{2h}\gamma_{hh}k^2 - \delta_{\lambda_{2h}} 2\lambda_{2h} + \frac{1}{8\pi^2\epsilon}\bigg[- 12y_h^2 m_\psi^2 - 8y_{2h}m_\psi^3 - 2y_h^2 k^2 \\ + 6g_{2A2h}m_A^2 + 6g_{2Ah}^2 + 3k^2\gamma^{\chi\chi}\left(\frac{g_{A\partial h\chi} + g_{A\partial\chi h}}{2}\right)^2 \\ + 12\gamma^{hh}\lambda_{4h}m_h^2 + 2\gamma^{\chi\chi}\lambda_{2h2\chi}m_\chi^2 + 18(\gamma^{hh})^2\lambda_{3h}^2 + 2(\gamma^{\chi\chi})^2\lambda_{h2\chi}^2\bigg]. \tag{46}$$

The γ^{aa} factors stem from the Higgs and GB propagators. Further, we used $m_h^2 = \gamma^{hh}(2\lambda_{2h})$ and similar for the GB mass. This yields

$$\delta_{\lambda_{2h}} = \frac{1}{8\pi^2\epsilon}\left[10s\lambda + 3\frac{g^4}{\lambda} - \frac{y^4}{\lambda}\right], \tag{47}$$

while for the kinetic term we retrieve (45).

Comparing with the CW result we find that $\delta_{\lambda_{2h}} = \delta_{V_0}$. In the large field regime this gives the equality $\delta_\lambda - \delta_\phi - 3\delta_\xi = \delta_\lambda - 2\delta_\xi$, from which we get

$$\delta_\xi = -\delta_\phi, \tag{48}$$

which assures that Ω does not run. This is the same as derived in the mid field regime from $\delta_{2h} = \mathcal{O}(\delta)$. In the small field regime ξ drops out of the Lagrangian at leading order, and no relation for δ_ξ can be derived at this order.

The correction to the GB propagator is

$$\Pi^\chi = -\delta_{2\chi}\gamma_{\chi\chi}k^2 - \delta_{\lambda_{2\chi}} 2\lambda_{2\chi} + \frac{1}{8\pi^2\epsilon}\bigg[- 4y_\chi^2 m_\psi^2 - 8y_{2\chi}m_\psi^3 - 2y_\chi^2 k^2 \\ + 6g_{2A2\chi}m_A^2 + 3k^2\gamma^{hh}\left(\frac{g_{A\partial h\chi} + g_{A\partial\chi h}}{2}\right)^2 \\ + 12\gamma^{\chi\chi}\lambda_{4\chi}m_\chi^2 + 2\gamma^{hh}\lambda_{2h2\chi}m_h^2 + 4\gamma^{hh}\gamma^{\chi\chi}\lambda_{h2\chi}^2\bigg]. \tag{49}$$

We find $\delta_{\lambda_{2h}} = \delta_{\lambda_{2\chi}}$ in all three regimes, as required by gauge invariance. Further we have

$$\delta_{2\chi} = \delta_\phi = \frac{1}{8\pi^2\epsilon}\left[3sg^2 - y^2\right]. \tag{50}$$

It is interesting to note that in the large field regime the counterterm $\delta_{\lambda 2\chi} = \mathcal{O}(\delta^3)$ whereas the individual fermion and gauge loop diagrams in (49) are $\mathcal{O}(\delta^2)$. Renormalizability thus requires the two fermion diagrams to cancel at leading order, to end up with an $\mathcal{O}(\delta^3)$ loop correction. This is indeed what happens. This intricate cancellation is even more pronounced

when we consider corrections to higher n-point GB scattering. For example, the structure of the fermion contribution to the four-point GB vertex is

$$
\begin{aligned}
V^{(4\chi)} =& -\delta_{\lambda 4\chi} 4! \lambda_{4\chi} \\
&+ \frac{4!}{8\pi^2\epsilon}\bigg[36\lambda_{4\chi}^2(\gamma^{\chi\chi})^2 + \lambda_{2h2\chi}^2(\gamma^{hh})^2 - m_h^2\lambda_{2h4\chi}\gamma^{hh} - 15m_\chi^2\lambda_{6\chi}\gamma^{\chi\chi} + 8\lambda_{h4\chi}\lambda_{h2\chi}\gamma^{hh}\gamma^{\chi\chi} \\
&\quad - 4y_{4\chi}m_\psi^3 - 4y_{3\chi}y_\chi(m_\psi^2 + \frac{1}{2}k^2) - 6y_{2\chi}^2(m_\psi^2 + \frac{1}{6}k^2) - 4y_{2\chi}y_\chi^2 m_\psi - y_\chi^4 \\
&\quad + 3(g_{2A2\chi}^2 + g_{2A4\chi}m_A^2)\bigg].
\end{aligned}
\tag{51}
$$

Now the counterterm on the first line is $\delta_{\lambda 4\chi} = \mathcal{O}(\delta^5)$. The GB and Higgs loop diagrams on the second line above give $\mathcal{O}(\delta^6)$ corrections and can be neglected. All individual fermion loop diagrams — the terms on the third line — and all individual gauge loop diagrams — the terms on the fourth line — are $\mathcal{O}(\delta^3)$, much larger than the counterterm. Thus both the leading and subleading contributions need to cancel when adding the diagrams. This is indeed what happens and we find $\delta_{\lambda_{2h}} = \delta_{\lambda_{4\chi}}$ as required by gauge invariance. This intricate cancellation, and the need to go to sub-sub-leading order in the δ-expansion, is the reason we cannot easily define the wavefunctions for the covariant fields, as discussed in section (3.4.1).

Note, however, that the k^2-term in (51) above does not cancel, and gives a correction that cannot be absorbed. [5] For this we have to add a new dimension-6 counterterm which is a four-point χ-interaction with two derivatives; very schematically

$$
\mathcal{L} \supset \frac{1}{\Lambda^2}\chi^2(\nabla\chi)^2, \tag{52}
$$

with a cutoff

$$
\Lambda \sim (y_{3\chi}y_\chi + \frac{1}{2}y_{2\chi}^2)^{-1/2} \gtrsim \Lambda_{\text{unitarity}} \tag{53}
$$

that is equal to or larger than the unitarity cutoff

$$
\Lambda_{\text{unitarity}} \sim \left\{\frac{1}{\xi}, \phi_0, \frac{1}{\sqrt{\xi}}\right\} \tag{54}
$$

in the small, mid and large field regime regime. The EFT breaks down for energy scales beyond the unitarity cutoff. The new counterterms needed to absorb divergencies enter at even higher scales. As such they do not put further constraints on the domain of validity of the EFT.

4.3 Yukawa interactions

We first calculate the corrections to the fermion propagator. The fermion-gauge coupling is standard over the whole field range, and the gauge loop gives the same result in all three regimes. This is not the case for the Higgs and GB loop, as the former is suppressed in the

[5] Here k is the momentum flowing in the loop at one of the vertices; the propagator structure is $\int \mathrm{d}^4 l D_\psi(l) D_\psi(l+k)$.

mid and large field regime. The fermion two-point function is

$$\Pi^{(2\psi)} = -\delta_\psi \not{k} - \delta_{m_\psi} m_\psi + \frac{1}{8\pi^2\epsilon}\Big[- 3m_\psi g^2 q_L q_R + y_h^2\gamma^{hh}(m_\psi - \frac{1}{2}\not{k}) - y_\chi^2\gamma^{\chi\chi}(m_\psi + \frac{1}{2}\not{k}) + y_{2h}\gamma^{hh}m_h^2 + y_{2\chi}\gamma^{\chi\chi}m_\chi^2\Big], \tag{55}$$

where we used $g_{A\bar\psi_{L,R}\psi_{L,R}} = g q_{L,R}$ in all three regimes. There is a minus sign difference between the two $\not{k}$ terms, which originates from the $(i\gamma^5)$ in vertices with an odd number of GBs. This gives for the counterterms

$$\delta_\psi = -\frac{1}{8\pi^2\epsilon}\left[\frac{y^2}{4}(s+1)\right],$$
$$\delta_{m_\psi} = \delta_y + \delta_\psi + \frac{1}{2}\delta_\phi = \frac{1}{8\pi^2\epsilon}\left(-3g^2 q_L q_R + \frac{1}{2}(s-1)y^2\right). \tag{56}$$

The GB and Higgs contribution add in δ_ψ and cancel in δ_{m_ψ} in the small field regime; in the mid and large field regime only the GB contribution survives.

The Yukawa interactions, both Higgs-fermion and GB-fermion, should give consistent results. Indeed we find

$$V_{\rm tot}^{\Psi\bar\Psi h} = -\frac{1}{8\pi^2\epsilon}\Big(-y_h^3\gamma^{hh} + y_h y_\chi^2\gamma^{\chi\chi} - 3m_h^2 y_{3h}\gamma^{hh} - m_\chi^2 y_{h2\chi}\gamma^{\chi\chi} - 2y_h y_{2h}\gamma^{hh}\left(\not{k} + 2m_\Psi\right) - \frac{y_\chi y_{h\chi}\gamma^{\chi\chi}}{2}\left(\not{k} - 2m_\Psi\right) - 6y_{2h}\lambda_{3h}(\gamma^{hh})^2 - 2y_{2\chi}\lambda_{h2\chi}(\gamma^{\chi\chi})^2 + 3q_L q_R g^2_{\bar\Psi A\Psi} y_h\Big) - \delta_{y_h} y_h, \tag{57}$$

and

$$V_{\rm tot}^{\Psi\bar\Psi\chi} = -\frac{i\gamma^5}{8\pi^2\epsilon}\left(y_h^2 y_\chi\gamma^{hh} - y_\chi^3\gamma^{\chi\chi} - m_h^2 y_{2h\chi}\gamma^{hh} - 3m_\chi^2 y_{3\chi}\gamma^{\chi\chi} - \frac{y_h y_{h\chi}\gamma^{hh}}{2}\left(\not{k} + 2m_\Psi\right) - 2y_\chi y_{2\chi}\gamma^{\chi\chi}\left(\not{k} + 2m_\Psi\right) - 2y_{h\chi}\lambda_{h2\chi}\gamma^{hh}\gamma^{\chi\chi} + 3q_L q_R g^2_{\bar\Psi A\Psi} y_\chi\right) - i\gamma_5\delta_{y_\chi} y_\chi. \tag{58}$$

This indeed gives $\delta_{y_h} = \delta_{y_\chi} = \delta_{m_\psi}$, with the latter given in (56).

However, once again there is a small glitch as the $\not{k}$ terms in both expressions do not cancel. New non-renormalizable counterterms need to be added, schematically of the form

$$\mathcal{L} \supset \frac{1}{\Lambda}\bar\psi(h + i\gamma^5\chi)\not{\partial}\psi \tag{59}$$

with cutoff

$$\Lambda \sim (\gamma^{\chi\chi} y_\chi y_{2\chi})^{-1} \gtrsim \Lambda_{\rm unitarity}. \tag{60}$$

Since the cutoff exceeds the unitarity cutoff, these terms do no affect the range of validity of the EFT in the three regimes.

4.4 Gauge interactions

We begin with the gauge boson propagator.

$$
\begin{aligned}
\Pi^A_{\mu\nu} &= -\delta_A(k^2 g_{\mu\nu} - k_\mu k_\nu) - \delta_{m_A} m_A^2 g_{\mu\nu} \\
&+ \frac{1}{8\pi^2\epsilon}\Bigg[\left(3\gamma^{hh} g^2_{h2A} + 2\gamma^{hh} g_{2h2A} m_h^2 + 2\gamma^{\chi\chi} g_{2\chi 2A} m_\chi^2\right) g^{\mu\nu} \\
&+ \gamma^{hh}\gamma^{\chi\chi}\left[-\frac{1}{4}(g_{A\partial h\chi} + g_{A\partial\chi h})^2\left(\frac{k^2}{3} + m_h^2 + m_\theta^2\right) g^{\mu\nu} + (g^2_{A\partial h\chi} - g_{A\partial h\chi} g_{A\partial\chi h} + g^2_{A\partial\chi h})\frac{1}{3}k^\mu k^\nu\right] \\
&- \frac{2}{3}(k^2 g^{\mu\nu} - k^\mu k^\nu)\left(g_L^2 + g_R^2\right) - 2(g_L - g_R)^2 m_\psi^2\Bigg]
\end{aligned}
\tag{61}
$$

It should be remembered that we normalized $q_\phi = 1$. The counterterms are

$$
\delta_A = -\frac{1}{8\pi^2\epsilon} g^2\left(s\frac{1}{3} + \frac{2}{3}q_L^2 + \frac{2}{3}q_R^2\right). \tag{62}
$$

Using the Ward identity $2\delta_g = -\delta_A$ (29), it follows that $\delta_{m_A} = 2\delta_g + \delta_\phi + \delta_A = \delta_\phi$. Reading off δ_{m_A} from the above expression, and comparing with our earlier result (50) for δ_ϕ, we indeed find agreement.

In the large field regime there is also a derivative interaction at leading order that is not transversal and cannot be absorbed in δ_A. We find a term

$$
\Pi^A \supset \frac{1}{8\pi^2\epsilon}\frac{g^2}{(1+6\xi)} k^\mu k^\mu. \tag{63}
$$

This term can be neglected only for $\xi \gg 1$. This is the only place where this extra condition is needed. The transverse term breaks the Ward identities in the Landau gauge, and should not be there. It arises as a consequence of our approximations, discussed in more detail in section 4.5. We are not too worried about this term, as it is absent in the large ξ limit. But moreover, it is also a gauge dependent term. We could have chosen a gauge fixing

$$
\mathcal{L}^E_{\rm GF} = -\frac{1}{2\xi_G}\left(\partial^\mu A_\mu - g\frac{\phi_0}{\Omega_0}\xi_G \chi\right)^2 \tag{64}
$$

defined in terms of the covariant fields rather than the Jordan frame fields (20). In Landau gauge, this gauge fixing gives the same results for all other diagrams, but now also the transversal part (63) vanishes.

As a consistency test we also calculated the $2A2h$ interaction, which gives

$$
\begin{aligned}
V^{2A2h} = \frac{g^{\mu\nu}}{8\pi^2\epsilon}\Big(&48\lambda_{4h} g_{2A2h} q_\Phi^2 (\gamma^{hh})^2 + 8\lambda_{2h2\chi} g_{2A2\chi}(\gamma^{\chi\chi})^2 - (g_{A\partial h\chi} + g_{A\partial\chi h})^2\gamma^{hh}(\gamma^{\chi\chi})^2\lambda_{2h2\chi} \\
&- 6(g_{A\partial h\chi} + g_{A\partial\chi h})^2(\gamma^{hh})^2\gamma^{\chi\chi}\lambda_{4h} + 24 g^2_{2A2h}\gamma^{hh} - 4(q_L - q_R)^2 y_h^2 g^2_{\bar\Psi A\Psi} \\
&- 8(q_L - q_R)^2 y_{2h} g^2_{\bar\Psi A\Psi} m_\Psi\Big) - 4\delta_{g_{2A2h}} g_{2A2h} g^{\mu\nu},
\end{aligned}
\tag{65}
$$

yielding $\delta_{m_A} = \delta_{g_{2A2h}}$, as it should.

4.5 Gauge-fermion vertex

There is one interaction that does not give a consistent result, which is the fermion-gauge coupling. To calculate it the important terms in the Lagrangian are

$$\mathcal{L} = -\frac{3\xi^2}{\Omega^4}\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi) - \frac{(D_\mu\Phi)^\dagger D^\mu\Phi}{\Omega^2} + \sum_{i=L,R} i\bar{\Psi}_i \not{D}\Psi_i - \frac{y\Phi\left(\bar{\Psi}_L\Psi_R + \text{h.c.}\right)}{\Omega}. \tag{66}$$

There are three one-loop diagrams: 1) a GB loop with the photon attached to the fermion line, 2) a Higgs loop with the photon attached to the fermion line, and 3) a mixed Higgs-GB loop with the photon attached via a derivative interaction to Φ. The result is

$$\begin{aligned} V_{\text{loop}}^{\bar{\Psi}A_\mu\Psi} &= -\frac{\gamma^\mu}{16\pi^2\epsilon}\Bigg[g_{\bar{\Psi}A\Psi}(y_\chi^2\gamma^{\chi\chi} + y_h^2\gamma^{hh})\,(q_L P_R + q_R P_L) \\ &\qquad + (g_{A\partial h\chi} + g_{A\partial\chi h})\, y_h y_\chi \gamma^{hh}\gamma^{\chi\chi}\,(q_\Phi P_L - q_\Phi P_R)\Bigg], \\ V_{\text{CT}}^{\bar{\Psi}A_\mu\Psi} &= -\delta_{\bar{\Psi}A_\mu\Psi}g_{\bar{\Psi}A)\mu\Psi}\gamma^\mu(q_L P_L + q_R P_R). \end{aligned} \tag{67}$$

In the small field regime this reduces to

$$V_{\text{loop}}^{\bar{\Psi}A_\mu\Psi} = -\frac{gy^2\gamma^\mu}{16\pi^2}\frac{1}{\epsilon}\left(q_L P_L + q_R P_R\right), \tag{68}$$

where we used gauge invariance: $q_\Phi - q_L + q_R = 0$. This expression can be absorbed in the counterterm, which is proportional to $(q_L P_L + q_R P_R)$ as well. In the large field regime, however, the diagrams with a Higgs loop are suppressed and we get

$$V_{\text{loop}}^{\bar{\Psi}A_\mu\Psi} = \frac{\gamma^\mu}{32\pi^2}\frac{1}{\epsilon}\left[\left(\frac{gy^2}{6\xi^3\phi_0^4} + gy^2\right)(q_L P_R + q_R P_L) + \frac{gy^2}{3\xi^3\phi_0^4}\left(q_\Phi P_L - q_\Phi P_R\right)\right]. \tag{69}$$

We cannot combine the two parts, and will not get something proportional to $(q_L P_L + q_R P_R)$. The same problem arises in the mid-field regime.

This result in the large field regime breaks gauge invariance explicitly. How did it arise? When we repeat the calculation without the first term in (66), the Higgs and GB field still have the same propagator. As a result all three diagrams contribute and the result adds up to something gauge invariant. However, when we include the first term, things go wrong as the Higgs propagator is now suppressed compared to the GB propagator. Note however, that the first term is explicitly gauge invariant. It is our approximation that breaks the gauge invariance, when we evaluate the metric on the background $\gamma_{ij}(\phi,\theta) = \gamma_{ij}(\phi_0)$. In particular, for the first term we set

$$\mathcal{L} \supset -\frac{3\xi^2}{\Omega^4}\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi) = -\frac{3\xi^2\phi_0^2}{2\Omega_0^4}(\partial_\mu\phi)^2 + \ldots \tag{70}$$

where the ellipses denote neglected higher order derivative interactions (to be precise: higher n-point interactions with two derivatives). We listed this as the third approximation in subsection 3.5. These higher order terms need to be included to obtain a gauge invariant result. Unfortunately, it does not seem straightforward to do so. We would like to postpone the setup of a framework able to handle higher order derivative terms to future work, leaving a loose thread to our current calculation. However, since the calculation of the two-point function involves lower order vertices, we expect it to be less prone to our approximation.

5 RGE equations

First we give the betafunctions for the Abelian-Higgs model with a non-minimal coupling, then in subsection 5.2 we generalize to full SM Higgs inflation.

5.1 Abelian Higgs model

First we list all the counterterms, found in the previous section:

$$\begin{aligned}
\delta_\phi = -\delta_\xi &= \frac{1}{8\pi^2\epsilon}\left(3g^2 s - y^2\right), \\
\delta_{\lambda_{2h}} = \delta_{\lambda_{4h}} = \delta_{\lambda_{2\chi}} = \delta_{\lambda_{4\chi}} &= \frac{1}{8\pi^2\epsilon}\left(10\lambda s + 3\frac{g^4}{\lambda} - \frac{y^4}{\lambda}\right), \\
\delta_\psi &= -\frac{1}{8\pi^2\epsilon}\left(\frac{y^2}{4}(s+1)\right), \\
\delta_{m_\psi} = \delta_{y_{h\bar{\psi}\psi}} = \delta_{y_{\chi\bar{\psi}\psi}} &= \frac{1}{8\pi^2\epsilon}\left(-3g^2 q_L q_R + \frac{1}{2}y^2(s-1)\right), \\
\delta_A &= -\frac{1}{8\pi^2\epsilon}g^2\left(\frac{1}{3}q_\phi^2 s + \frac{2}{3}q_L^2 + \frac{2}{3}q_R^2\right).
\end{aligned} \tag{71}$$

The counterterms for the couplings in the Lagrangian are then given by $\delta_\lambda = \delta_{\lambda_{4h}} - 2\delta_\phi$, $\delta_y = \delta_{m_\psi} - 1/2\delta_\phi - \delta_\psi$, and the Ward identiy $\delta_g = -1/2\delta_A$ respectively. From this the beta-functions can be found via $\beta_\lambda = \lambda(\epsilon\delta_\lambda)$, and similarly for the Yukawa, gauge and non-minimal Higgs-gravity coupling. This gives

$$\begin{aligned}
\beta_\lambda &= \frac{1}{8\pi^2}\left(10s\lambda^2 + 3g^4 - y^4 - 6sg^2\lambda + 2y^2\lambda\right), \\
\beta_y &= \frac{1}{8\pi^2}\left(-3q_L q_R g^2 y - \frac{3}{2}s(q_L - q_R)^2 g^2 y + \frac{1}{4}(1+3s)y^3\right) \\
\beta_g &= \frac{1}{8\pi^2}g^3\left(s\frac{1}{6}q_\phi^2 + \frac{1}{3}q_L^2 + \frac{1}{3}q_R^2\right), \\
\beta_\xi\big|_{\text{mid,large}} &= -\frac{1}{8\pi^2}(s3g^2 - y^2)\xi.
\end{aligned} \tag{72}$$

We have not derived β_ξ in the small field regime. At leading order all ξ dependence drops out of the Lagrangian in the SM regime. Since our computation relies on the approximations listed in section 3.5, which fail beyond the leading order, we have to leave the computation of β_ξ in the small field regime open.

5.2 SM Higgs inflation

Our results for a U(1) theory can be extended to the full Standard Model (SM) Higgs inflation. Working in background field gauge [6], the symmetries of the classical effective action are similar to those of the U(1) theory. The main difference is that now there are 3 GBs, the top quark has three colors, and one needs to sum over the strong, weak and hypercharge interactions.

[6] In practice, it is not so easy to calculate diagrams in the background field gauge, as it is unclear how to expand the Lagrangian in covariant fields with a shifted metric.

Higgs coupling First we extend the U(1) results in the small field regime to the full SM beta-functions, which can be found for example in [40]. The SM betafunction for the Higgs self-coupling is a straightforward generalization of the U(1) result:

$$\begin{aligned}\beta_\lambda &= \frac{1}{8\pi^2}\left[(9+n_\theta)\lambda^2 + 3\sum_a g_a^4 - n_c y_t^4 - 2\lambda(3\sum_a g_a^2 - n_c y_t^2)\right] \\ &= \frac{1}{8\pi^2}\left[12\lambda^2 + \frac{3}{16}\left(2g_2^4 + (g_2^2+g_1^2)^2\right) - 3y_t^4 - 2\lambda(\frac{9}{4}g_2^2 + \frac{3}{4}g_1^2 - 3y_t^2)\right], \end{aligned} \tag{73}$$

with $n_\theta = 3$ the number of GBs, $n_c = 3$ the number of colors and the g_a are given in (13). We have only included the running of the top Yukawa. Generalizing from the U(1) model, it follows that the λ^2 and the $\lambda g_{1,2}^2$ terms are suppressed in the mid and large field regime. This gives

$$\beta_\lambda = \frac{1}{(4\pi)^2}\left[24\lambda^2 s + \frac{3}{8}\left(2g_2^4 + (g_2^2+g_1^2)^2\right) - 6y_t^4 - \lambda(9g_2^2 + 3{g_1}^2)s + 12y_t^2\lambda)\right]. \tag{74}$$

Gauge coupling For an $SU(N)$ group the betafunction is

$$\beta(g)\Big|_{SU(N)} = -\frac{g^3}{96\pi^2}\left(22N - 2n_f - n_H\right), \tag{75}$$

with n_f the number of Weyl fermions and n_s the number of complex Higgs fields, both in the fundamental representation. For an Abelian group there is no contribution from the gauge field, and the formula becomes

$$\beta(g)\Big|_{U(N)} = \frac{g^3}{96\pi^2}\left(4\sum q_f^2 + \sum q_s^2\right), \tag{76}$$

with q_f, q_s the charges of the Weyl fermion and real scalars respectively. This reproduces our result in the small field regime. In the mid and large field the Higgs and GB contributions to the running are absent; this does not affect QCD, but for the EW sector we get

$$\begin{aligned}\beta_{g_3} &= -\frac{7}{(4\pi)^2}g_3^3, \\ \beta_{g_2} &= -\frac{1}{(4\pi)^2}\frac{(20-s)}{6}g_2^3, \\ \beta_{g_1} &= \frac{1}{(4\pi)^2}\frac{(40+s)}{6}g_1^3.\end{aligned}$$

Yukawa coupling The running of the top Yukawa follows from the counterterm $\delta_y = \delta_{y_{h\bar\psi\psi}} - 1/2(\delta_\phi + \delta_{t_L} + \delta_{t_R})$. Explicit expressions can e.g. be found in [41, 42]. For the top quark the SM counterterms are:

$$\begin{aligned}\delta_{t_L} &= -\frac{1}{8\pi^2\epsilon}\left(\frac{s}{4}y_t^2 + \frac{1}{4}y_t^2 + \frac{1}{2}y_b^2\right), \\ \delta_{t_R} &= -\frac{1}{8\pi^2\epsilon}\left(\frac{s}{4}y_t^2 + \frac{1}{4}y_t^2 + \frac{1}{2}y_t^2\right).\end{aligned} \tag{77}$$

The y^2 contributions stem from loops with h, χ and $\phi^+ = \chi_1 - i\chi_2$ respectively. For t_L the ϕ^+ loop can only be made with a bottom quark in the loop, which gives a contribution to y_b^2 — which we neglect in the following. In our U(1) toy model we only had the first two contributions from the h and χ-loop; indeed this matches the counterterm we found before. In the mid and large field regime the Higgs loop is suppressed, but not the GB loop. Likewise, we expect the ϕ^+-loop to contribute in the large field regime, as these are GBs, with the same structure of interactions as χ.

The Higgs counterterm is

$$\delta_\phi = -\frac{1}{8\pi^2\epsilon}\left[n_c y_t^2 - 3s\sum_a g_a^2\right] = -\frac{1}{8\pi^2\epsilon}\left[3y_t^2 - \frac{3}{4}s(3g_2^2 + g_1^2)\right], \tag{78}$$

which generalizes our previous results. Here n_c is the number of colors. The gauge contribution is suppressed in the large field regime.

The vertex correction is

$$\delta_{y_{h\bar{\psi}\psi}} = -\frac{1}{8\pi^2\epsilon}\left[-\frac{1}{2}y_t^2(s-1) + 3Y_{tL}Y_{tR}g_1^2 + 3C_2(R_t)g_3^2\right], \tag{79}$$

with $C_2(R_t) = 4/3$ for the fundamental in SU(3), and $Y_{tL} = 1/6$, $Y_{tR} = 2/3$. For the U(1) model we found that the y^2-correction from the h and χ-loop cancels in the small field regime. However, in the large field regime the Higgs contribution is negligible, and there is a net contribution from the GB χ. The ϕ^+ loop gives a contribution $\propto y_b^2$ and can be neglected. The gauge terms stem from top-top-gauge loops, and since the fermion-gauge couplings are standard in the large field regime, these are unaffected.

This gives for the betafunction

$$\beta_{y_t} = \frac{1}{(4\pi)^2}\left[\frac{3}{2}(2+s)y_t^3 - \left(\frac{8+9s}{12}g_1^2 + \frac{9s}{4}g_2^2 + 8g_3^2\right)y_t\right]. \tag{80}$$

In the small field regime $s = 1$ and we get the standard result.

Non-minimal coupling Further, we found in the large and mid field regime that $\delta_\phi = -\delta_\xi$. This gives the betafunction for the non-minimal coupling

$$\beta_\xi\big|_{\text{mid,large}} = \frac{1}{(4\pi)^2}\left[6y^2 - \frac{3}{2}s(3g_2^2 + g_1^2)\right]\xi. \tag{81}$$

5.2.1 End results

$$
\begin{aligned}
(4\pi)^2\beta_\lambda &= 24\lambda^2 s + A + (4\pi)^2 \cdot 4\lambda\gamma_\phi \\
(4\pi)^2\gamma_\phi &= -\frac{s}{4}(3g_1^2 + 9g_2^2) + 3y_t^2 \\
(4\pi)^2\beta_{g_3} &= -7g_3^3, \\
(4\pi)^2\beta_{g_2} &= -\frac{(20-s)}{6}g_2^3, \\
(4\pi)^2\beta_{g_1} &= \frac{(40+s)}{6}g_1^3 \\
(4\pi)^2\beta_{y_t} &= \frac{3}{2}sy_t^3 - \left(\frac{2}{3}g_1^2 + 8g_3^2\right) y_t + (4\pi)^2 \cdot \gamma_\phi y_t \\
(4\pi)^2\beta_\xi\big|_{\text{mid,large}} &= (4\pi)^2 \cdot 2\gamma_\phi \xi
\end{aligned} \tag{82}
$$

with $A = (3/8)(2g_2^4 + (g_2^2 + g_1^2)^2) - 6y_t^4$.

6 Discussion

6.1 Comparison with literature.

In recent years several groups have presented renormalization group equations for SM Higgs inflation. The disagreement between these results has been a major motivation to write this paper which follows, in our opinion, the most systematic approach so far. In this section we compare our findings to some encountered in the recent literature.

Let us first quickly compare this work to our own previous work [27]. There we have studied the renormalization of just a (complex) non-minimally coupled scalar, leaving the inclusion of fermions and gauge fields and the generalization to full SM Higgs inflation to this work. Our findings here generalize those in [27]. Note in particular that in that work, we concluded that we could not say anything about the RG flow in the mid-field regime, as the corrections were an order of δ smaller than the counterterms. However, the fermionic and some of the gauge corrections that we have found now are of the same order as the counterterms. That is why we can now present expressions for the running couplings in the mid field regime (barring threshold corrections) without getting in contradiction with our previous work.

Now for the comparison to other authors. In general, it seems that all other approaches follow some predefined treatment for Higgs and Goldstone bosons. In some cases only the Higgs contributions are kept, in other cases only the GBs, in yet other cases only loop contributions are excluded, etcetera. Our result does not respect any of these guidelines. For example, we find that the GB contribution to the effective potential is suppressed, while the GB loops do contribute to the Yukawa corrections. To see exactly which field contributes to which loop correction all correction diagrams need to be properly computed. (Although the differences are never very dramatic.)

We first compare to reference [5], which states that in large field the action is just the action of the chiral SM with $v = m_{\rm p}/\sqrt{\xi}$, and the Higgs (but not the GBs) decouples. The

RGEs quoted for the large field regime are

$$\begin{aligned}
(4\pi)^2\beta_\lambda &= A + (4\pi)^2 \cdot 4\lambda\gamma_\phi \\
(4\pi)^2\gamma_\phi &= -\frac{1}{4}(3g_1^2 + 6g_2^2) + 3y_t^2 \\
(4\pi)^2\beta_{g_2} &= -\frac{(20 - 1/2)}{6}g_2^3, \\
(4\pi)^2\beta_{g_1} &= \frac{(40 + 1/2)}{6}g_1^3 \\
(4\pi)^2\beta_{y_t} &= -\left(\frac{2}{3}g_1^2 + 8g_3^2\right) y_t + (4\pi)^2 \cdot \gamma_\phi y_t \\
(4\pi)^2\beta_\xi &= (4\pi)^2 \cdot 2\gamma_\phi\xi.
\end{aligned} \tag{83}$$

Here a gauge contribution in γ_ϕ has been included, which explains the difference in β_λ, β_ξ and β_{y_t} with our work. In the betafunctions for the gauge coupling only the real Higgs field is excluded, whereas we also exclude the GB.

The chiral SM is non-renormalizable, and new operators have to be included. At one-loop level there is a correction to the Z-boson mass, which depends on the running coefficients of two of these new operators. There is no such thing in our set-up, which is renormalizable in the EFT sense.

We furthermore note that in this work the running of ξ is computed with an approach very different from ours, via the running of the SM vev v. However, apart from our disagreement over the gauge contribution to γ_ϕ, we find the same answer.

Reference [23] states that in the inflationary regime quantum loops involving the Higgs field are heavily suppressed. The proposed prescription (originally introduced in [1]) is to assign one factor of $s(\phi)$ for every off-shell Higgs that runs in a quantum loop, with

$$s(\phi) = \frac{1}{\Omega^2}\gamma_{\phi\phi} = \frac{1 + \xi\phi_0^2}{1 + (6\xi + 1)\xi\phi_0^2}. \tag{84}$$

Although the metric factor in s above is for the real Higgs field, judging from the RGEs presented the prescription has been applied to the complex field (nothing is said explicitly about Higgs and GBs). In large field, the quoted RGEs reduce to:

$$\begin{aligned}
(4\pi)^2\beta_\lambda &= A + (4\pi)^2 \cdot 4\lambda\gamma_\phi \\
(4\pi)^2\gamma_\phi &= -\frac{1}{4}(3g_1^2 + 9g_2^2) + 3y_t^2 \\
(4\pi)^2\beta_{g_2} &= -\frac{20}{6}g_2^3, \\
(4\pi)^2\beta_{g_1} &= \frac{40}{6}g_1^3 \\
(4\pi)^2\beta_{y_t} &= -3y_t^3 - \left(\frac{2}{3}g_1^2 + 8g_3^2\right) y_t + (4\pi)^2 \cdot \gamma_\phi y_t \\
(4\pi)^2\beta_\xi &= 2((4\pi)^2 \cdot \gamma_\phi + 6\lambda)(\xi + 1/6).
\end{aligned} \tag{85}$$

There is a gauge contribution to γ_ϕ, which explains the difference in β_λ, β_ξ and partly β_{y_t} with our result. In β_{g_i} the full Higgs doublet is taken out in the large field regime, in agreement with

our result. β_ξ is found by taking gravity as a classical background, following the pioneering work in [43, 44, 45]. We think that this is not a good approximation in the Jordan frame.

This same s-factor formalism was followed in References [46, 47], with the modification that now only the loops of the real Higgs field are excluded, and not those of the GBs. However our final answers agree with neither of the results obtained there.

Reference [26] writes that Goldstone modes, in contrast to the Higgs particle, do not have mixing with gravitons in the kinetic term. Therefore, their contribution is not suppressed by the s-factor. We disagree with this. The GBs cannot be treated as usual, as in polar coordinates $\rho^2(\partial\theta)^2$ the radial field is not the canonical one. In Cartesian coordinates, all fields are equally coupled to the Ricci tensor via $\xi R\sum(\phi^i)^2$. The quoted results are

$$
\begin{aligned}
(4\pi)^2\beta_\lambda &= 6\lambda^2 + A - (4\pi)^2\cdot 4\lambda\gamma_\phi \\
(4\pi)^2\gamma_\phi &= \frac{1}{4}(3g_1^2+9g_2^2) - 3y_t^2 \\
(4\pi)^2\beta_{g_2} &= -\frac{(20-1/2)}{6}g_2^3, \\
(4\pi)^2\beta_{g_1} &= \frac{(40+1/2)}{6}g_1^3 \\
(4\pi)^2\beta_{y_t} &= \left[y_t^3 - \left(\frac{2}{3}g_1^2+8g_3^2\right)y_t\right] - (4\pi)^2\cdot\gamma_\phi y_t \\
(4\pi)^2\beta_\xi &= 6\xi\lambda - (4\pi)^2\cdot 2\gamma_\phi\xi.
\end{aligned}
\tag{86}
$$

A gauge contribution to γ_ϕ is included, which partly explains the difference in β_λ, β_ξ and β_{y_t} with our results. For β_{y_t} the contribution of one GB y_t^2-term has been excluded instead of 3GB y_t^2-terms. In β_{g_i} only the GB is taken out in the large field regime, in disagreement with our result.

This should be the identical to the chiral model of [5], as the Higgs field is decoupled in the large field limit. However the RGEs are still different.

Lastly, [5, 23] use two different normalization conditions, one with a field independent cutoff in the Jordan frame, or with a field dependent cutoff in the Jordan frame. However, two frames give identical physics. It is often quoted that a field independent cutoff in the Jordan frame corresponds to a field dependent cutoff in the Einstein frame, and vice versa. However, dimensionful quantities by themselves have no invariant meaning, their values depend on the unit system. If we express the cutoff in Planck units (the Planck mass is frame dependent), a constant cutoff in the one frame is equivalent to a constant cutoff in the other frame. The conformal rescaling only rescales *all* length scales, which does not change the physics. See also our discussion in [27]. On-shell equivalence between the frames has also been established in [48].

The question about a field dependent or independent cutoff is a frame invariant question when expressed in Planck units. The choice of cutoff has thus nothing to do with a choice of frame. One can still debate whether the results depend on the (field dependent) choice of cutoff in the Einstein frame. A priori, this is not expected; the cutoff is only introduced to regularize the divergent integrals, but is at the end taken to infinity. Requiring the counterterms to be field independent, the different field dependent and independent cutoffs lead to different normalization conditions. In practice one can only relate physical measurements at different energy scales. The translation between the observable and the coupling defined in the normalization condition will be different in each case. The end result is that when

comparing physical observables at different scales, the cutoff dependence drops out.

6.2 Conclusions

We have calculated the one-loop corrections to Higgs inflation in the small, mid and large field regime. We have done the calculations for the Abelian Higgs model; the results can then rather straightforwardly be generalized to full Standard Model Higgs inflation. We have found that in all three regimes the model is renormalizable in the effective field theory sense. The RGEs for SM Higgs inflation we found are given in (82). The results for the mid field regime are new. The running of the non-minimal coupling can be derived in the mid and large field regime, and follows from the consistency of the radiative corrections to the potential and to the two-point functions. In the small field regime all dependence on the non-minimal coupling drops out of the equations at leading order in the small field expansion, and nothing can be said about its running.

The computation of the radiative corrections was done in the Einstein frame, in the Landau gauge, using a covariant formalism for the multi-field system. The one-loop corrections to the propagators are sufficient to determine the full set of counterterms, and thus the betafunctions. As extra checks, we have calculated many higher n-point functions as well. Especially the results for four-point scattering of the Goldstone bosons are impressive in this regard: in the large (and mid) field regime both the leading and subleading divergencies exactly cancel, yielding a consistent counterterm.

However, we have stumbled on some potential problematic outcomes as well. First of all, to cancel all divergencies, new non-renormalizable counterterms need to be added, see (53, 60). However, the cutoff implied by these new counterterms always exceeds the unitarity cutoff (54). Therefore they do not put further constraints on the validity of the EFTs in the various regimes. Second, in (63) we have seen that the gauge boson propagator picks up a transverse part, that should be absent by the Ward identities. This term vanishes in the large ξ limit. Moreover, it is gauge dependent, and we believe it should vanish in a full calculation. Thirdly, the one-loop gauge-fermion vertex gives a gauge symmetry breaking result as well. We can trace it back to the explicit symmetry breaking in our approximation of the non-minimal kinetic terms. In a full calculation the symmetry should be restored, yielding a result consistent with the gauge propagator corrections that we have found, but we leave this for further work.

In conclusion, we have computed the full set of RGE equations for Standard Model Higgs inflation. The Higgs-fermion part has withstood an impressive set of consistency checks. When including the gauge symmetry our result obtained from propagator corrections has failed one consistency test, which we think can be ascribed to the intrinsic limitations in our approach (neglecting higher order kinetic terms by evaluating the field metric on the background). It would be an interesting but equally challenging task to develop a framework that can get around these limitations.

Acknowledgments

DG is funded by a Herchel Smith fellowship. SM is supported by the Fondecyt 2015 Post-doctoral Grant 3150126 and by the "Anillo" project ACT1122, funded by the "Programa de Investigación Asociativa." MP is funded by the Netherlands Foundation for Fundamental

Research of Matter (FOM) and the Netherlands Organisation for Scientific Research (NWO). We thank Mikhail Shaposhnikov and Sergey Sibiryakov for illuminating discussions.

A Couplings

We list the couplings for the Abelian U(1) model. The explicit values in the small, mid, and large field regime are found expanding in δ_i:

$$\begin{aligned} &\delta_s = \xi\phi_0, && \text{small} \\ &\xi \to \delta_m^{-2}\xi,\ \phi_0 \to \delta_m^{3/2}\phi_0, && \text{mid} \\ &\delta_l = 1/(\xi\phi_0^2). && \text{large} \end{aligned} \tag{87}$$

Note that contrary to the small and large field regime, the δ_m parameter is just a rescaling parameter. In the small field we express the results in ϕ_0 rather than δ_s, as this form is more familiar. Below we give the leading expression for the metric and for the relevant couplings; the three values between the braces correspond to the small, mid and large field regime.

The metric is:

$$\begin{aligned} \gamma_{hh} &= \left\{1,\ \frac{6\phi_0^2\xi^2}{\delta_m},\ (6\xi+1)\delta_l\right\} \\ \gamma_{\chi\chi} &= \{1,\ 1,\ \delta_l\}\,. \end{aligned} \tag{88}$$

The Higgs and GB self-interactions are

$$\begin{aligned} \lambda_{2h} &= \frac{1}{2!}V_{;\phi\phi}\Big|_{\rm bg} = \lambda\left\{\frac{3}{2}\phi_0^2,\ \phi_0^2\delta_m^3,\ -\frac{1}{\xi}\delta_l^2\right\} \\ \lambda_{2\chi} &= \frac{1}{2!}V_{;\theta\theta}\Big|_{\rm bg} = \lambda\left\{\frac{1}{2}\phi_0^2,\ \frac{1}{12\xi^2}\delta_m^4,\ \frac{\delta_l^3}{2\xi(1+6\xi)}\right\} \\ \lambda_{3h} &= \frac{1}{3!}V_{;\phi\phi\phi}\Big|_{\rm bg} = \lambda\left\{\phi_0,\ \left(\frac{1}{18\phi_0\xi^2} - 2\phi_0^3\xi\right)\delta_m^{5/2},\ \frac{2\delta_l^{5/2}}{3\sqrt{\xi}}\right\} \\ \lambda_{h2\chi} &= \frac{1}{3!}\left(V_{;\phi\theta\theta} + V_{;\theta\phi\theta} + V_{;\theta\theta\phi}\right)\Big|_{\rm bg} = \lambda\left\{\phi_0,\ \frac{\delta_m^{5/2}}{18\phi_0\xi^2},\ -\frac{4\delta_l^{7/2}}{3\sqrt{\xi}(1+6\xi)}\right\} \\ \lambda_{4h} &= \frac{1}{4!}V_{;\phi\phi\phi\phi}\Big|_{\rm bg} = \lambda\left\{\frac{1}{4},\ -\frac{\delta_m}{18\phi_0^2\xi^2},\ -\frac{\delta_l^3}{3}\right\} \\ \lambda_{2h2\chi} &= \frac{1}{4!}\left(V_{;\phi\phi\theta\theta} + 5\text{perms}\right)\Big|_{\rm bg} = \lambda\left\{\frac{1}{2},\ -\frac{\delta_m}{18\phi_0^2\xi^2},\ \frac{11\delta_l^4}{6(1+6\xi)}\right\} \\ \lambda_{4\chi} &= \frac{1}{4!}V_{;\theta\theta\theta\theta}\Big|_{\rm bg} = \lambda\left\{\frac{1}{4},\ \frac{\delta_m^2}{432\phi_0^4\xi^4},\ -\frac{\delta_l^5}{3(1+6\xi)^2}\right\} \\ \lambda_{5h} &= \text{etc.} \end{aligned} \tag{89}$$

The Yukawa interactions are

$$
\begin{aligned}
m_\psi &= F^\phi\Big|_{\rm bg} = \frac{y}{\sqrt{2}}\left\{\phi_0,\ \phi_0\delta_m^{3/2},\ \frac{1}{\sqrt{\xi}}\right\} \\
y_h &= F^\phi_{;\phi}\Big|_{\rm bg} = \frac{y}{\sqrt{2}}\left\{1,\ 1,\ \delta_l^{3/2}\right\} \\
y_{2h} &= \frac{1}{2!}F^\phi_{;\phi\phi}\Big|_{\rm bg} = \frac{y}{\sqrt{2}}\left\{-(3\xi^2+\xi)\phi_0,\ \ -\frac{1}{2\phi_0\delta_m^{3/2}}, -\sqrt{\xi}\delta_l^2\right\} \\
y_{3h} &= \frac{1}{3!}F^\phi_{;\phi\phi\phi}\Big|_{\rm bg} = \frac{y}{\sqrt{2}}\left\{-\frac{1}{3}(3\xi^2+\xi),\ \ -\frac{1}{2\phi_0^2\delta_m^3},\ \frac{2}{3}\xi\delta_l^{5/2}\right\} \\
y_{4h} &= \text{etc.} \\
y_\chi &= F^\theta_{;\theta}\Big|_{\rm bg} = \frac{y}{\sqrt{2}}\left\{1,\ 1,\ \sqrt{\delta_l}\right\} \\
y_{2\chi} &= \frac{1}{2!}F^\phi_{;\theta\theta}\Big|_{\rm bg} = \frac{y}{\sqrt{2}}\left\{-(3\xi^2+\xi)\phi_0,\ \ -\frac{1}{2\phi_0\delta_m^{3/2}},\ \ -\frac{1}{2}\sqrt{\xi}\delta_l\right\} \\
y_{3\chi} &= \frac{1}{3!}F^\theta_{;\theta\theta\theta}\Big|_{\rm bg} = \frac{y}{\sqrt{2}}\left\{-\frac{1}{3}(3\xi^2+\xi),\ \ -\frac{1}{6\phi_0^2\delta_m^3},\ \ -\frac{1}{6}\xi\delta_l^{3/2}\right\} \\
y_{4\chi} &= \frac{1}{4!}F^\phi_{;\theta\theta\theta\theta}\Big|_{\rm bg} = \frac{y}{\sqrt{2}}\left\{\left(\frac{\xi^2}{3}+3\xi^3+\frac{15\xi^4}{2}\right)\phi_0,\ \frac{1}{24\phi_0^3\delta_m^{9/2}},\ \frac{1}{24}\xi^{3/2}\delta_l^2\right\}
\end{aligned}
\tag{90}
$$

with

$$
F^\phi = \frac{y}{\sqrt{2}}\frac{\phi_0+\varphi}{\Omega}, \qquad F^\theta = \frac{y}{\sqrt{2}}\frac{\theta}{\Omega},
\tag{91}
$$

as follows from (15).

Finally, the gauge interactions are

$$
\begin{aligned}
m_A^2 &= g^2 G\Big|_{\rm bg} = g^2\left\{\ \phi_0^2,\ \phi_0^2\delta_m^3,\ \frac{1}{\xi}\right\} \\
g_{2Ah} &= g^2 G_{;\phi}\Big|_{\rm bg} = g^2\left\{\ \phi_0,\ \phi_0\delta_m^{3/2},\ \frac{\delta_l^{3/2}}{\sqrt{\xi}}\right\} \\
g_{2A2h} &= \frac{1}{2!}g^2 G_{;\phi\phi}\Big|_{\rm bg} = g^2\left\{\ \frac{1}{2},\ \left(\frac{1}{12\phi_0^2\xi^2}-\phi_0^2\xi\right)\delta_m,\ \ -\delta_l^2\right\} \\
g_{2A2\chi} &= \frac{1}{2!}g^2 G_{;\theta\theta}\Big|_{\rm bg} = g^2\left\{\ \frac{1}{2},\ \frac{\delta_m}{12\phi_0^2\xi^2},\ \frac{\delta_l^3}{2(1+6\xi)}\right\} \\
g_{A\chi\partial h} &= gG^\phi_{;\theta}\Big|_{\rm bg} = g\left\{1,\ 1,\ \delta_l\right\} \\
g_{Ah\partial\chi} &= gG^\theta_{;\phi}\Big|_{\rm bg} = g\left\{1,\ 1,\ -\delta_l\right\} \\
g_{A\bar\psi_{L,R}\psi_{L,R}} &= gq_{L,R}
\end{aligned}
\tag{92}
$$

with G, G^ϕ, G^θ defined in (15). To get the two derivative couplings, we have simply set $\partial\phi = \partial h$ and $\partial\theta = \partial\chi$. For higher derivative couplings (to be precise: with still one derivative but more fields, such as $g_{A2h\partial\chi}$) we need to go beyond the first terms in these expansions, by using (19).

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